Notes

Physics Tool box

- **Electric Potential Energy** – the electric potential energy stored in a system of two charges q_1 and q_2 is $U_E = \frac{kq_1q_2}{r}$
- **k** – Coulombs Constant is $\frac{1}{4\pi \in_0} = 9.0\times10^9 N\cdot\frac{m^2}{C^2}$
- **Electric Potential** –A distance ***r*** from a charge ***q*** is given by $V = \frac{U_E}{q_2} = \frac{kq_1}{r}$ where q_2 is the test charge and q_1 is the charge creating the electric field. This is measured in volts.
- **Voltage** – a common term for electric potential : 1 volt is the electric potential at a point in an electric field if 1 J of work is required to move 1 C of charge from infinity to that point: $1V = 1\frac{J}{C}$.
- The ***Potential Difference*** between two points in an electric field is given by the change in the electric potential energy of a **positive charge** as it moves from one point to another: $\Delta V = \frac{\Delta U_E}{q}$.
- The magnitude of the electric field is the change in potential difference per unit radius: $\varepsilon = \frac{\Delta V}{r}$

Here we are going to talk about energy associated with electrical interactions. Every time you turn on an electrical appliance, you are making use of electrical energy. We are already familiar with the concepts of work and energy in the context of mechanics; now we will combine these concepts with what we have learned about electric charges, electric force, and electric fields.

When a charged particle moves in an electric field, the field exerts a force that can do work on the particle. This work can always be expressed in terms of electric potential energy. Just as gravitational potential energy depends on the height of the mass above the Earth's surface, electric potential energy depends upon the position of the charged particle in the electric field.

Recall:

The magnitude of the force of gravity between two masses is given by:

$$F_G = \frac{Gm_1m_2}{r^2}$$

The gravitational potential energy between two masses is given by:

$$U_G = -\frac{Gm_1m_2}{r}$$

Now consider a small test charge q_2, a distance r from a point charge q_1. From Coulomb's Law, the magnitude of the force of attraction or repulsion between these two charges is:

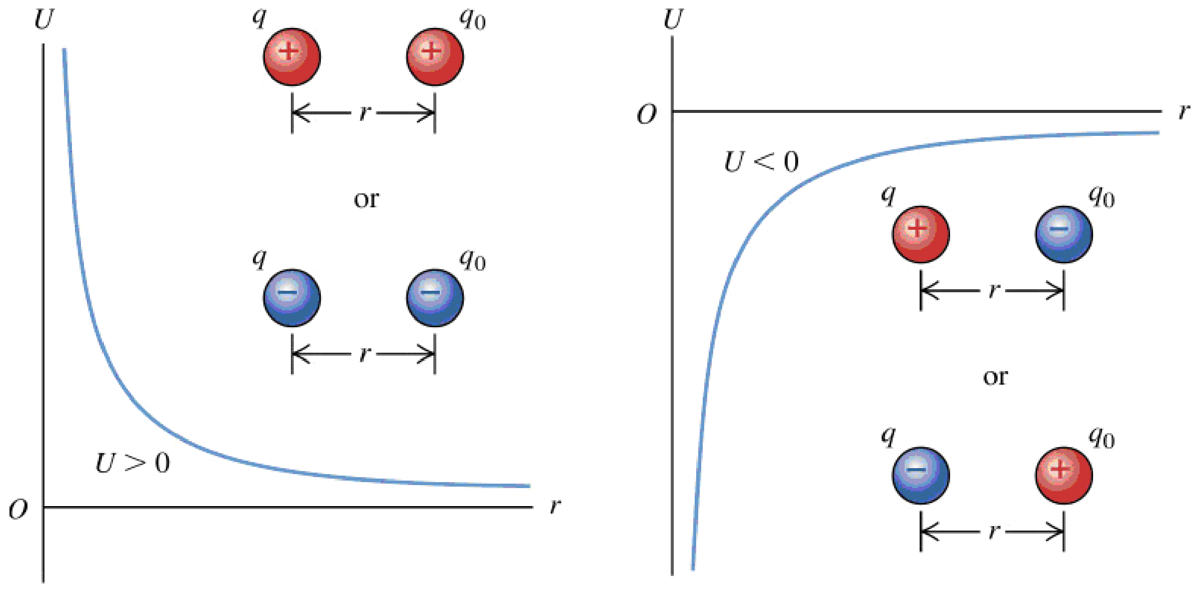
$$F_E = \frac{kq_1q_2}{r^2}$$

Therefore, it seems reasonable that an approach similar to what we did in the previous note that the following would be the result for **electric potential energy**:

$$U_G = \frac{kq_1q_2}{r}$$

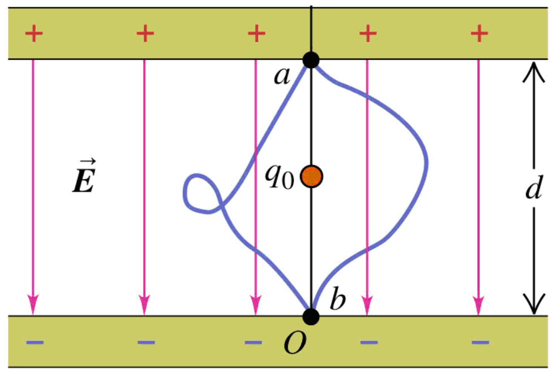
Note: the is no negative sign on the potential energy equation since it could be positive or negative. If both charges have same sign (ie repel) then the potential energy will be positive. If the charges have opposite signs (ie will attraction) then the potential energy will be negative.

We see that Potential energy increases as opposite charges recede, and that Potential energy increases as like charges approach.



(a) q, q_0 have same sign (b) q, q_0 have opposite signs

Electric potential or voltage is the difference in potential energy of a charge that moves from one point in an electric field to another. Electric potential is similar to height in gravity problems. At the same height above Earth's surface, the potential energy of all objects is converted to kinetic energy to produce the same final speed. The objects actual energy value depends on its mass. Similarly, a charge gains energy by moving in a electric field through a given voltage difference. The amount of energy it gains depends on how much charge it has.



Regardless of the actual path taken by the charge q_0 in moving a to b.

$$\left\{\begin{array}{l}\text{the difference between}\\ \text{the electric potential}\\ \text{at b and the electric}\\ \text{potential at a}\end{array}\right\} = \left\{\begin{array}{l}\text{the work-per-unit-charge that}\\ \text{we would perform in moving our}\\ \text{hypothetical positive test charge}\\ \text{from a to b in the electric field}\end{array}\right\}$$

$$\begin{aligned}\Delta U_E &= q_0V_b - q_0V_a\\ &= q_0\left(V_b - V_a\right)\\ &= q_0\Delta V\end{aligned}$$

Note: ΔV is usually written as V_{ba}

For a point charge q_0, the electric potential difference between point b and a can be found by subtracting the electric potentials due to the charge at each position.

$$V_{ba} = \Delta V = V_b - V_a = \frac{kq_0}{r_b} - \frac{kq_0}{r_a} = kq_0\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

Now multiplying by the charge that is moved from a to b gives us the change in electric potential ΔU.

$$\Delta U = q\Delta V$$

For a given position of the test charge in the field, the ***charge–field*** system has a potential energy ***U*** relative to the configuration of the system that is defined as U = 0. Dividing this potential energy, ***U***, by the value of the test charge, q_o gives a physical quantity that depends only on the source charge distribution. The potential energy per unit charge $\frac{U}{q_o}$ is independent of the value of q_o and has a value at every point in an electric field. This Quantity $\frac{U}{q_o}$ is called the ***electric potential*** (or simply the ***potential***) ***V***.

Thus, the electric potential at any point in an electric field is:

$$V = \frac{U}{q_o}$$

The fact that potential energy is a scalar quantity means that electric potential also is a scalar. If the test charge is moved between two positions ***A*** and ***B*** in an electric field, the charge–field system experiences a change in potential energy. The ***potential difference*** $\Delta V = V_B - V_A$ between two points ***A*** and ***B*** in an electric field is defined as the change in potential energy of the system when a test charge is moved between the points divided by the test charge q_0:

$$\Delta V \equiv \frac{\Delta U}{q_o} = -\int_A^B \vec{\varepsilon} \cdot d\vec{s}$$

Just as with potential energy, only differences in electric potential are meaningful. To avoid having to work with potential differences, however, we often take the value of the electric potential to be zero at some convenient point in an electric field. Potential difference should not be confused with difference in potential energy. The potential difference between ***A*** and ***B*** depends only on the source charge distribution (consider points ***A*** and ***B*** without the presence of the test charge), while the difference in potential energy exists only if a test charge is moved between the points. Electric potential is a scalar characteristic of an electric field, independent of any charges that may be placed in the field.

> **Note:** *The **potential** is characteristic of the field only, independent of a charged test particle that may be placed in the field. **Potential energy** is characteristic of the charge–field system due to an interaction between the field and a charged particle placed in the field.*

Typically we use variety of phrases to describe the potential difference between two points, the most common being ***voltage***, arising from the unit for potential. A voltage applied to a device, such as a computer, or across a device is the same as the potential difference across the device. If we say that the voltage applied to a computer is 12 volts, we mean that the potential difference between the two electrical contacts on the computer is 12 volts.

Comparison of Electric Potential Energy and Electric Potential

In order to bring two ***like*** charges (positive and positive or negative and negative) near each other work must be done. Whenever work gets done, energy changes form. When the one charge is brought closer to the other charge, a positive amount of work would be done by you. This work would increase the ***electrical potential energy*** of the system.

The formula for ***electric potential energy*** is: $U_G = \frac{kq_1q_2}{r}$

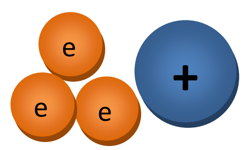
Since the ***electrical potential energy*** can change depending on the amount of charge you are moving, it is useful to describe the ***electrical potential energy*** per unit of charge. This is known as ***electrical potential***

*NOTE: the name sounds very similar to **electrical potential energy**, but the role is different.*

$$\text{Electric Potential} = \frac{\text{Work done on Particle}}{\text{Unit of Charge Moved}} = \frac{\Delta\ \text{Electric Potential Energy}}{\text{Unit of Charge Moved}}$$

The formula is usually written as: $V = \frac{W}{q_{moved}} = \frac{kQ_{not\ moved}}{r}$ with units of $\frac{J}{C}$ *or Volt*

For Example:



You do 30J of work to pull apart 3 positive charges (1 coulomb each) from a negative charge. This amount of work increases the ***electric potential energy*** of the 3 positive charges (these charges retain 30J worth of ***Electric Potential Energy***). The ***electric potentia***l (not energy) is the amount of energy per Unit Charge.

$$V = \frac{W}{q_{moved}} = \frac{30J}{3C} = 10\frac{J}{C} = 10\ Volts$$

At the original position, when the 3 positive charges are next to the negative charge, the charges they have no ***potential energy***, so they also have no ***electrical potential*** or in other words V=0 volts. But, once they are pulled apart, they then have an ***electrical potential*** of 10 volts. You should notice that the ***electrical potential difference*** from the one point to the other is 10 volts.

Remember that the ***electrical potential*** describes the amount of energy per unit of charge. This means that if you release one of the charges then the electric field will do 10 Joules of work on the charge so it will have a kinetic energy of 10 Joules the instant before it strikes the negative charge.

In your world of experience, A 9 Volt battery indicates that every Coulomb 6.241×10^{18}) of charge that moves from the negative side of the battery to the positive side will do 9 Joules worth of work.

Electric Plates

The increase in electric potential energy of the charge q, in moving from plate B to plate A, is equal to the work done in moving it from B to A. To do so, a force $\vec{F}$, equal in magnitude but opposite in direction to $\vec{F}_E$, must be applied over a distance r.

The magnitude of the work done is:

$$\begin{aligned} W &= \nabla U_E \\ &= q\Delta V \end{aligned}$$

$$\begin{aligned} W &= F \cdot r \\ &= F_E r \\ &= q\varepsilon r \\ \Delta U_E &= q\varepsilon r \\ q\Delta V &= q\varepsilon r \\ \varepsilon &= \frac{\Delta V}{r} \end{aligned}$$

Potential Energy question

Calculate the closest distance an alpha particle with charge $3.2 \times 10^{-19} C$ gets to a gold nucleus with charge $1.26 \times 10^{-17} C$ if the alpha particle has $6.1 \times 10^{-13} J$ of kinetic energy.

Solution:

This question involves the law of conservation of energy. All of the alpha particle's kinetic energy is converted to potential energy by the time it is stopped by the repulsive force between the two positive charges

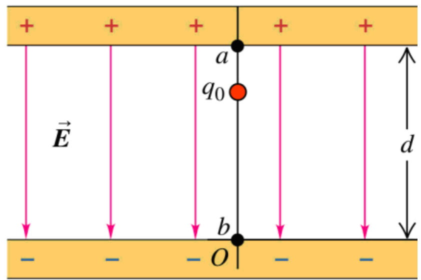
$$\begin{aligned} E_{T_i} &= E_{T_f} \\ E_{ki} + U_i &= E_{kf} + U_f \\ 6.1 \times 10^{-13} J + 0J &= 0J + \frac{kq_1q_2}{r} \\ r &= \frac{kq_1q_2}{6.1 \times 10^{-13} J} \\ &= \frac{\left(9.0 \times 10^9 \frac{N \cdot m^2}{C^2}\right)\left(3.2 \times 10^{-19} C\right)\left(1.26 \times 10^{-17} C\right)}{6.1 \times 10^{-13} J} \\ &= 5.94885 \times 10^{-14} m \end{aligned}$$

The closest distance is $5.9 \times 10^{-14} m$

Potential difference Question

The potential difference between Positive charge plate A and negative charged plate B is 40 V.

a) Which plate is at the higher potential?
b) How much work must be done to carry a 3.0 C charge from B to A
c) How much work must be done to carry a 3.0 C charge from A to b
d) How do we know that the electric field goes from A to B
e) If the plate separation is 5.0 mm, what is the magnitude of the electric field?



Solution:

a) A positive test charge between the plates is repelled by A and attracted by B. Left to itself, the positive test charge will move from A to b, and so A is at the higher potential.

b) The magnitude of the work done in carrying a charge q through a potential difference V is W=qV

$$\begin{aligned} W &= qV \\ &= (3.0C)(40V) \\ &= 120J \end{aligned}$$

Because a positive charge between the plates is repelled by A, positive work (+120J) must be done to drag the +3.0C charge from B to A.

c) Since the +3.0 C charge will want to move form A to B, then -120J of work is done moving it from A to B.

d) A positive test charge between the plates experiences a force directed from A to B and this is, by definition, the direction of the field.

e) For parallel plates : $V = \varepsilon r$

$$\begin{aligned} \varepsilon &= \frac{\Delta V}{r} \\ &= \frac{40V}{0.0050m} \\ &= 8000\frac{V}{m} \end{aligned}$$

The magnitude of the electric field strength is 8000 V/m or 8000 N/C (recall that the SI units for electric field V/m and N/C are identical)

Electric Potential Problem

Calculate the electric potential a distance of 0.50 m from a spherical point charge of $7.3\times10^{-6}C$ (take V=0 at infinity)

Solution:

$$
\begin{aligned}
V &= \frac{kq}{r} \\
&= \frac{\left(9.0\times10^{9}\frac{N\cdot m^{2}}{C^{2}}\right)\left(7.3\times10^{-6}C\right)}{0.50m} \\
&= 1.314\times10^{5}V
\end{aligned}
$$

The electric potential is $1.3\times10^{5}V$

Work Problem

How much work must be done to increase the potential of a charge $5.0\times10^{-7}C$ by 100V?

Solution:

$$
\begin{aligned}
W &= \Delta U_{E} \\
&= q\Delta V \\
&= \left(5.0\times10^{-7}C\right)\left(100V\right) \\
&= 5.0\times10^{-5}J
\end{aligned}
$$

Work Problem

Find the speed of a mass of $3.2\times10^{-6}kg$ if it moves through a potential difference of 40.0 V and has a charge of $5.1\mu C$.

Solution:

The equation $W = q\Delta V$ gives you the energy the charge gains in the form of kinetic energy. Therefore we can substitute $\frac{1}{2}mv^{2}$ for W and solve for the speed.

$$W = q\Delta V$$

$$\frac{1}{2}mv^2 = q\Delta V$$

$$v^2 = \frac{2q\Delta V}{m}$$

$$v = \sqrt{\frac{2q\Delta V}{m}}$$

$$= \sqrt{\frac{2\left(5.1\times10^{-6}c\right)\left(40.0V\right)}{3.2\times10^{-6}kg}}$$

$$= 11.29159\frac{m}{s}$$

The speed of the mass is therefore 11 m/s.

Example

The electric potential difference between the inside and outside of a neuron cell membrane of thickness 5.0 nm is typically 0.070V

a) Explain why the inner and outer surfaces of the membrane can be thought of as opposite charged parallel plates

b) Calculate the magnitude of the electric field in the membranes.

c) Calculate the work you would have to do on a single sodium ion, of charge $1.6\times10^{-19}C$, to move it through the membrane from the region of lower potential into the region of higher potential.

Solution:

a) Look at figure 13 on page 356. You notice a positively layer of extracellular fluid parallel to a negatively charged layer of intracellular fluid. These two parallel layers are separated by a cell membrane given the appearance of a set of parallel plates.

b) Since we are assuming the surfaces are acting as parallel plates. Then

$$\varepsilon = \frac{\Delta V}{d}$$

$$= \frac{0.070V}{5.0\times10^{-9}m}$$

$$= 1.4\times10^{7}\frac{N}{C}$$

c) If the electrical force moves a charge a certain distance, it does work on that charge. The change in ***electric potential*** over this distance is defined through the work done by this force:

Work done = (Force) x (distance) = (Potential difference) x (Charge)

$$\begin{aligned} W &= V \cdot q \\ &= (0.070V)(1.6\times10^{-19}C) \\ &= 1.1\times10^{-20}J \end{aligned}$$

Extra Notes and Comments