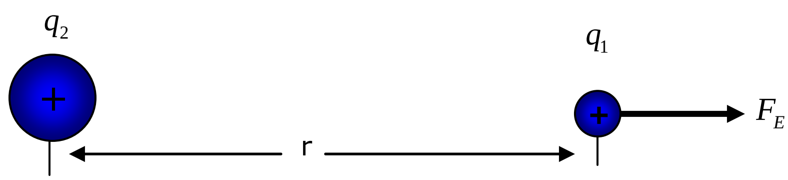
7.6 The Motion of charged Particles in electric Fields

Physics Tool box

- A charged particle in a uniform electric field moves with uniform motion.
- From conservation principles, any changes to a particle's kinetic energy result from corresponding changes to its electric potential energy (with respect to electric field, ignoring gravitational effects)



Given two particles, q_1 and q_2. Then the charge q_1 experiences a Coulomb force, to the right whose magnitude is given by:

$$F_E = \frac{kq_1q_2}{r}$$

If mass q_1 is free to move from its original position, it will accelerate in the direction of the electric force (Newton's 2nd Law) with an instantaneous acceleration whose magnitude is given by:

$$a = \frac{F_E}{m}$$

Now describing the subsequent motion of q_1 becomes difficult because as it begins to move, the distance r increases causing F_E to decrease, so the acceleration a will also decrease. This decrease in acceleration is a difficult analytical problem if we have only Newton's Laws to work with.

So we need to apply the properties of energies to assist us in solving these types of problems.

When q_1 is at r_1 then q_1 is at rest and so the total energy of the charged mass equals its potential energy

$$E_{T_1} = E_{T_2}$$
$$U_{E_1} + E_{K_1} = U_{E_2} + E_{K_2}$$
$$\frac{kq_1q_2}{r_1} + \frac{1}{2}mv_1^2 = \frac{kq_1q_2}{r_2} + \frac{1}{2}mv_2^2$$

$$\frac{kq_1q_2}{r_1}-\frac{kq_1q_2}{r_2}=\frac{1}{2}mv_2^2-\frac{1}{2}mv_1^2$$
$$-\left(\frac{kq_1q_2}{r_2}-\frac{kq_1q_2}{r_1}\right)=\frac{1}{2}mv_2^2$$

The charged particle q_1 moves in the electric field of q_2 in such a way that the electric potential energy it loses is equal to the kinetic energy it gains.

Example

Two small conducting spheres are placed on top of insulating pads. The $6.0\times10^{-10}C$ sphere is fixed while the $2.0\times10^{-7}C$ sphere is free to move. The mass of each sphere and pad is 0.12 kg. If the spheres are initially 0.15m apart, how fast will the sphere be moving when they are 1.0 m apart?

Solution:

$$\frac{kq_1q_2}{r_1}+\frac{1}{2}mv_1^2=\frac{kq_1q_2}{r_2}+\frac{1}{2}mv_2^2$$
$$\frac{kq_1q_2}{r_1}=\frac{kq_1q_2}{r_2}+\frac{1}{2}mv_2^2$$
$$v_2^2=\frac{2}{m}\left(\frac{kq_1q_2}{r_1}-\frac{kq_1q_2}{r_2}\right)$$
$$v_2=\sqrt{\frac{2}{m}\left(\frac{kq_1q_2}{r_1}-\frac{kq_1q_2}{r_2}\right)}$$
$$=\sqrt{\frac{2}{0.12kg}\left(\frac{\left(9.0\times10^{9}\frac{N\cdot m^2}{C^2}\right)\left(2.0\times10^{-7}C\right)\left(6.0\times10^{-10}C\right)}{0.15m}\left(\frac{1}{0.15m}-\frac{1}{1m}\right)\right)}$$
$$=\sqrt{0.000102\frac{m^2}{s^2}}$$
$$=0.0101\frac{m}{s}$$

The sphere will be moving with a speed of 0.010 m/s

Work

The work done by a constant force in the same direction as the displacement is the scalar product of the force and the displacement. In a parallel plate appartus with plate displacement r, the work done by the electric force in moving a charge q from one plate to another is

$$
\begin{aligned}
W &= \vec{F}_E \cdot \vec{r} \\
&= \varepsilon q r \\
&= \frac{\Delta V}{d} q r \\
&= \Delta V q
\end{aligned}
$$

Example

A proton (charge $+e = 1.602 \times 10^{-19} C$) moves in a straight line from point A to B inside a linear accelerator, a total distance of 0.40 m. The electric field is uniform along this line, with magnitude $\varepsilon = 1.2 \times 10^{7} \frac{V}{m} = 1.2 \times 10^{7} \frac{N}{C}$ in the direction from A to B. Determine

a) the force on the proton
b) the work done on it by the field
c) the potential difference $V_A - V_B$

Solution:

a) The force is in the same direction as the electric field, and its magnitude is

$$
\begin{aligned}
F &= q\varepsilon \\
&= \left(1.602 \times 10^{-19} C\right)\left(1.2 \times 10^{7} \frac{N}{C}\right) \\
&= 1.9224 \times 10^{-12} N
\end{aligned}
$$

The force is $1.9 \times 10^{-12} N$

b) The force is constant and in the same direction as the displacement, so the work done is

$$
\begin{aligned}
W_{A \to B} &= F \cdot d \\
&= \left(1.9224 \times 10^{-12} N\right)\left(0.40 m\right) \\
&= 7.6896 \times 10^{-13} J
\end{aligned}
$$

The work done is $7.7 \times 10^{-13} J$

c) $$\Delta V = \frac{W}{q}$$

$$
\begin{aligned}
V_A - V_B &= \frac{7.6806 \times 10^{-13} J}{1.602 \times 10^{-19} C} \\
&= 4.8 \times 10^{6} V
\end{aligned}
$$

The potential difference is $4.8 \times 10^{6} V$