Notes

Physics Tool box

- A current can exert a force on a magnet, and a magnet can exert a force on a current.
- $F_M = |q| v_\perp B = |q| v B \sin(\phi)$
- $\vec{F} = q\vec{v} \times \vec{B}$
- The direction of the magnetic force is given by the right-hand rule.
- To explore an unknown magnetic field, we can measure the magnitude and direction of the force on a moving test charge, then calculate B. The electron beam in a cathode-ray tube is a convenient device for making such measurements.

If you place a compass needle next to a wire (align the wire so that the needle points in the same direction as the wire). Now connect the wire to a DC power supply, the compass needle will then turn so that it is perpendicular to the direction of the wire (current).

Therefore, a current can exert a force on a magnet.

Two wires, side-by-side, both with a current passing through them will either attract or repel each other.

Therefore, a magnetic field can exert a force on a current

Magnetic Fields

Recall:

1. A distribution of electric charge at rest creates an electric field $\vec{E}$ in the surrounding space.
2. The electric field exerts a force $\vec{F} = q\vec{E}$ on any other charge q that is present in the field.

We can describe magnetic interactions in a similar way

1. A moving charge or current creates a magnetic field in the surrounding space (in addition to its electric field)
2. The magnetic field exerts a force $\vec{F}$ on any other moving charge or current that is present in the field.

Like the electric field, the magnetic field is a vector field – that is, a vector quantity associated with each point in space. We use the symbol $\vec{B}$ for magnetic field. At any position the direction of $\vec{B}$ is defined as that in which the north pole of a compass needle tend to point. (that is why the south geographic pole is the north magnetic pole).

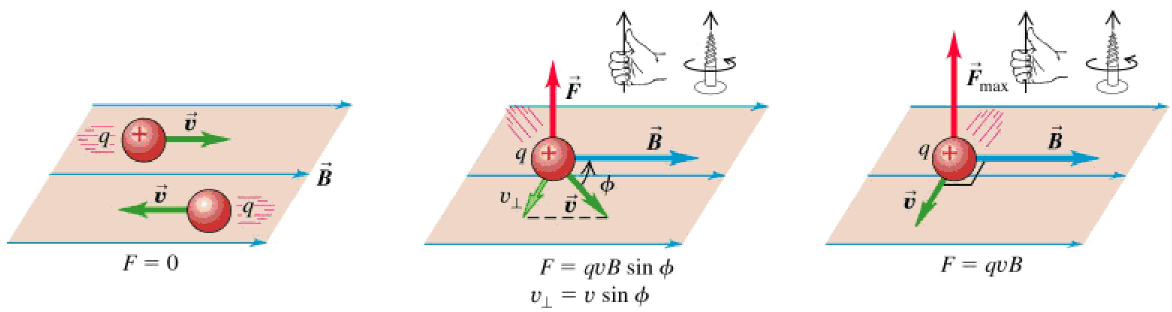
The magnitude of the magnetic force $\vec{F}_M$ on a charged particle has two factors that affect it:

- Is directly proportional to the magnitude of the magnetic field $\vec{B}$, the velocity $\vec{v}$, and the charge q of the particle.
- Depends on the angle ϕ between the magnetic field $\vec{B}$ and the velocity $\vec{v}$. When $\phi = 90°$ (particle is moving perpendicular to the field lines), the force is at a maximum, and when $\phi = 0°$ or $\phi = 180°$ (particle is moving parallel to the field lines) the force vanishes.

The magnitude F of the force is found to be proportional to the component of the velocity , $\vec{v}$, perpendicular to the field.

$$F = |q| v_{\perp} B = |q| vB\sin(\phi)$$

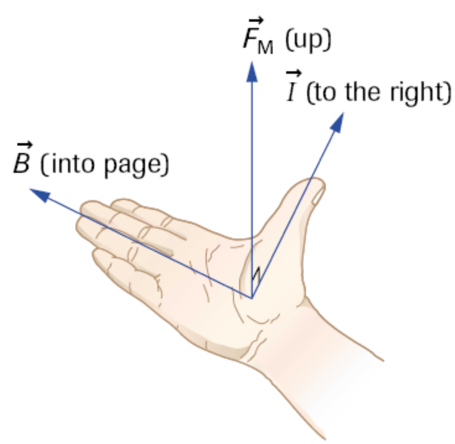
Where $|q|$ is the magnitude of the charge, and ϕ is the angle measured from the direction of $\vec{v}$ to the direction $\vec{B}$.



(a) Velocity $\vec{v}$ parallel or antiparallel to magnetic field $\vec{B}$: magnetic force is zero

(b) $\vec{v}$ at an angle ϕ to $\vec{B}$: magnetic force has magnitude $F = qvB \sin \phi$

(c) $\vec{v}$ perpendicular to $\vec{B}$: magnetic force has magnitude $F = qvB$



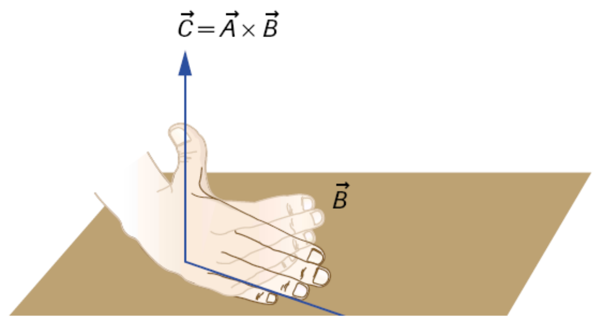
A simple right hand rule can be used to determine the direction of the force as follows: if the thumb pints in the direction of the current, $\vec{v}$ (positive charges), and the extended fingers point in the direction of the magnetic field, $\vec{B}$, the force is in the direction in which the palm points.

Optional

The vector or Cross Product of Two Vectors

The vector product of two vectors has a magnitude equal to the product of the magnitudes of the two vectors and the sine of the angle between the vectors. The cross product $\vec{C}$ of vectors $\vec{A}$ and $\vec{B}$, is written as $\vec{C} = \vec{A} \times \vec{B}$ where the magnitude of is given by $C = \left|\vec{C}\right| = AB\sin(\phi)$. The direct of C is perpendicular to the plane formed by $\vec{A}$ and $\vec{B}$. How ever there are two distinct direction that are perpendicular to that plane. Use another version of the right hand rule..

The right hand rule for cross products: when the fingers of the right hand curl from $\vec{A}$ to $\vec{B}$, the out-stretched thumb points in the direction of $\vec{C}$

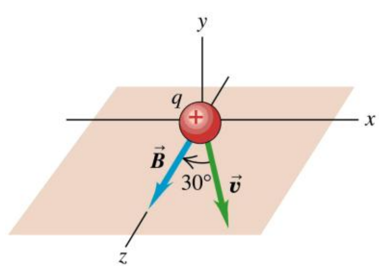


Example

A beam of protons $\left(q = 1.6\times10^{-19} C\right)$ moves at $3.0\times10^{5}\frac{m}{s}$ through a uniform magnetic field with magnitude 3.0 T that is directed along the z-axis. The velocity of each proton lies in the xz-plane at an angle of 30° to the +z-axis. Determine the force on a proton.

Solution:

The charge is positive, so the force is in the same direction as the vector product $\vec{v} \times \vec{B}$ which is in the negative y-axis.



$$\begin{aligned} F &= qvB\sin(\phi) \\ &= \left(1.6\times10^{-19}C\right)\left(3.0\times10^{5}\frac{m}{s}\right)(3.0T)\left(\sin(30^\circ)\right) \\ &= 7.2\times10^{-14}N \end{aligned}$$

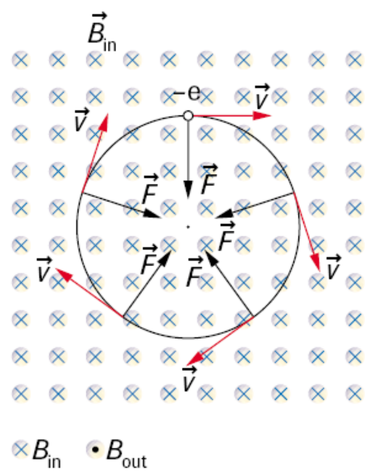
The magnitude of the force is $7.2\times10^{-14}N$ long the negative y-axis.

Note: if the charges were electrons (negative q), the force is oppose to $\vec{v}\times\vec{B}$

Example

An electron accelerates from rest in a horizontally directed electric field through a potential difference of 52 V. The electron then leaves the electric field, entering a magnetic field of magnitude 0.30 T directed into the page.

a. Calculate the initial speed of the electron upon entering the magnetic field.
b. Calculate the magnitude and direction of the magnetic force on the electron.
c. Calculate the radius of the electrons circular path.



Because the moving particles are electrons (negatively charged), the direction of the force causes the electrons to move in a curved path.

a. The electric potential energy lost by the electron in moving through the electric potential difference equals its gain in kinetic energy.

$$-\Delta U_E = \Delta E_K$$
$$q\Delta V = \frac{1}{2}mv^2$$
$$v = \sqrt{\frac{2q\Delta V}{m}}$$
$$= \sqrt{\frac{2\left(1.6\times10^{-19}C\right)\left(52V\right)}{9.11\times10^{-31}kg}}$$
$$= 4.3\times10^{6}\frac{m}{s}$$

b. $F_M = qvB\sin\left(\phi\right)$

$$= \left(1.6\times10^{-19}C\right)\left(4.3\times10^{6}\frac{m}{s}\right)\left(0.30\frac{kg}{C\cdot s}\right)\sin\left(90°\right)$$
$$= 2.1\times10^{-13}N$$

Applying the right hand rule with electrons if the electron is moving to the right, the force is down.

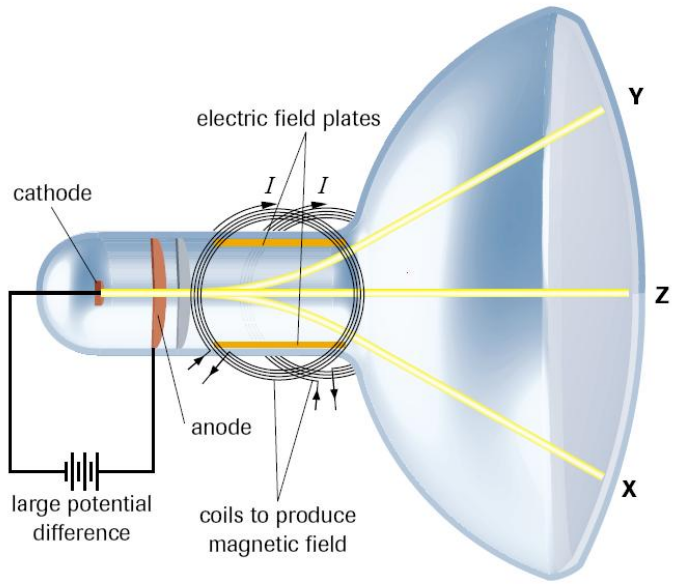
c. The electron will go in a uniform circular motion. The magnetic force is the net centripetal force.

$$F_M = F_C$$
$$qvB\sin\left(90°\right) = \frac{mv^2}{r}$$
$$r = \frac{mv}{Bq}$$
$$= \frac{\left(9.11\times10^{-31}kg\right)\left(4.3\times10^{6}\frac{m}{s}\right)}{\left(0.30T\right)\left(1.6\times10^{-19}C\right)}$$
$$= 8.2\times10^{-5}m$$

The radius of the circular path is $8.2\times10^{-5}m$.

Charge to Mass Ratios

The beam of electrons was called a cathode ray because the electron had not yet been discovered. A British scientist J.J. Thomson used two parallel plates, a cathode and an anode to generate a thin beam of electrons that got deflected by coils (produced magnetic fields), or other plates (produces electric fields) which causes the electron beam to de deflected up, down, left, or right. The cathode move from the cathode to the anode.



When there is current in the coils, it produces a magnetic field of magnitude B, which deflects the electrons along a circular arc of radius r so they hit the end of the tube at point X.

Now we know:

$$F_M = F_c$$
$$evB\sin(90°) = \frac{mv^2}{r}$$
$$\frac{e}{m} = \frac{v}{Br}$$

B can be calculated since we know the dimensions of the coils and the amount of current, and r can be measured directly.

Now v can by applying a potential difference between the two parallel plates (the lowe plate is negative) and turning off the current in the coils, the electron is deflected upward to reach tube at Y. now by adjusting the magnetic field and electric field so that the electron beam hits at Z (electric and magnetic field cancel), thus

$$F_M = F_E$$
$$evB\sin(90°) = e\varepsilon$$
$$v = \frac{\varepsilon}{B}$$

Where ε is the magnetic of the electric field between the parallel plates, $\varepsilon = \frac{V}{d}$.

Now using these equations, the ratio of charge to mass for electrons can be determined

$$\begin{aligned}\frac{e}{m} &= \frac{v}{Br} \\ &= \frac{\varepsilon}{B^2 r} \\ &= 1.76\times10^{11}\frac{C}{kg}\end{aligned}$$

A few years later, Millikan, determined that the charge on an electron is $1.60\times10^{-19}C$. Therefore the electron mass is $9.11\times10^{-31}kg$.

This technique can be used to determine the charge to mass ratio for any charged particle moving through known electric and magnetic fields.

$$\frac{q}{m} = \frac{\varepsilon}{B^2 r}$$

Example

Determine a formula for calculating the mass of a particle of charge q, through a potential difference of ΔV, into a uniform magnetic field of B when the radius of the curved path of the deflected mass is r.

Solution:

From $F_M = F_c$ and $\Delta E_c = \Delta E_K$

$$\begin{aligned} F_M &= F_C & \text{and} \quad \Delta E_K &= \Delta E_C \\ qvB &= \frac{mv^2}{r} & \frac{1}{2}mv^2 &= q\Delta V \\ v &= \frac{qBr}{m} & v &= \sqrt{\frac{2q\Delta V}{m}} \end{aligned}$$

Equating these

$$\begin{aligned}\frac{qBr}{m} &= \sqrt{\frac{2q\Delta V}{m}} \\ \frac{q^2B^2r^2}{m^2} &= \frac{2q\Delta V}{m} \\ m &= \frac{qB^2r^2}{2\Delta V}\end{aligned}$$

<u>Extra Notes and Comments</u>