8.4 Ampere's Law

Physics Tool box

- **Ampere's Law:** Along any closed path through a magnetic field, the sum of the products of the scalar component of $\vec{B}$, parallel to the path segment with the length of the segment, is directly proportional to the net electric current passing through the area enclosed by the path.
- $\sum B_{\parallel}\Delta l = \mu_0 I$
- SI defines an ampere as the current in each of two long, straight, parallel conductors 1 m apart in a vacuum, when the magnetic force between them is $2\times10^{-7}N$ per metre of length.
- SI defines the coulomb as the charge transported by a current of 1 A in a time of 1 s.

So far we have been discussing the force on a conductor with a current or on a moving charged particle, we have been dealing with a uniform magnetic field (magnetic fields constant in both direction and magnitude). However, magnetic field lines are not uniform. The question we will discuss is how can we determine the strength of a non-uniform magnetic field in a given point?

The Straight Conductor

Recall that the magnetic field around a straight conductor consists of field lines that are concentric circles, centred on the conductor (right hand rule). The circles become more widely spaced as the distance from the conductor increases. Measurement show that:

$$B \propto I$$
$$B \propto \frac{1}{r}$$

Therefore the magnitude of the magnetic field strength can be written as : $B = k\frac{I}{r}$

Where k is the proportionality constant (the direction of B via right hand rule)

Ampere found a general relationship between the current in any conductor (not just straight) and the strength of the magnetic field it produces.

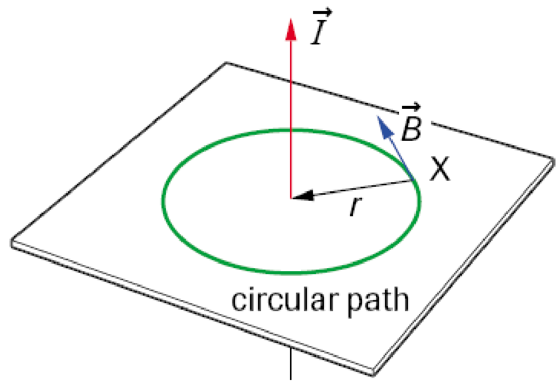
Ampere's Law

Along any closed path through a magnetic field, the sum of the products of the scalar component of $\vec{B}$, parallel to the path segment with the length of the segment, is directly proportional to the net electric current passing through the area enclosed by the path.

In mathematical terms: $\sum B_{\parallel}\Delta l = \mu_0 I$

Let's apply Ampere's law to a long straight conductor carrying a current I.

So we shall calculate the magnitude of the magnetic field strength at a point X, a distance r from the wire. We shall choose a circle for a path passing through X (since the law applies for any closed path, why not make a simple circular closed path).



Since a circular path centred around the current is used, we can make the simplification that $B_{\parallel} = B$ =constant, since the magnitude of the magnetic field around a current carrying wire is constant in magnitude at a constant distance from the wire because it is circular. Also $\vec{B}$ is parallel to Δl, since Δl is tangent to the circle (i.e, $\vec{B}$ has only one component, $B_{\parallel}$). Therefore

$$\sum B_{\parallel}\Delta l = \sum B\Delta l = B\sum \Delta l = B(2\pi r) = \mu_0 I$$

$$B = \mu_0 \left(\frac{I}{2\pi r}\right)$$

Example

What is the magnitude of a field 3.2 cm from a straight long conductor carrying a current of 4.1 A?

Solution:

$$B = \mu_0 \left(\frac{I}{2\pi r}\right)$$

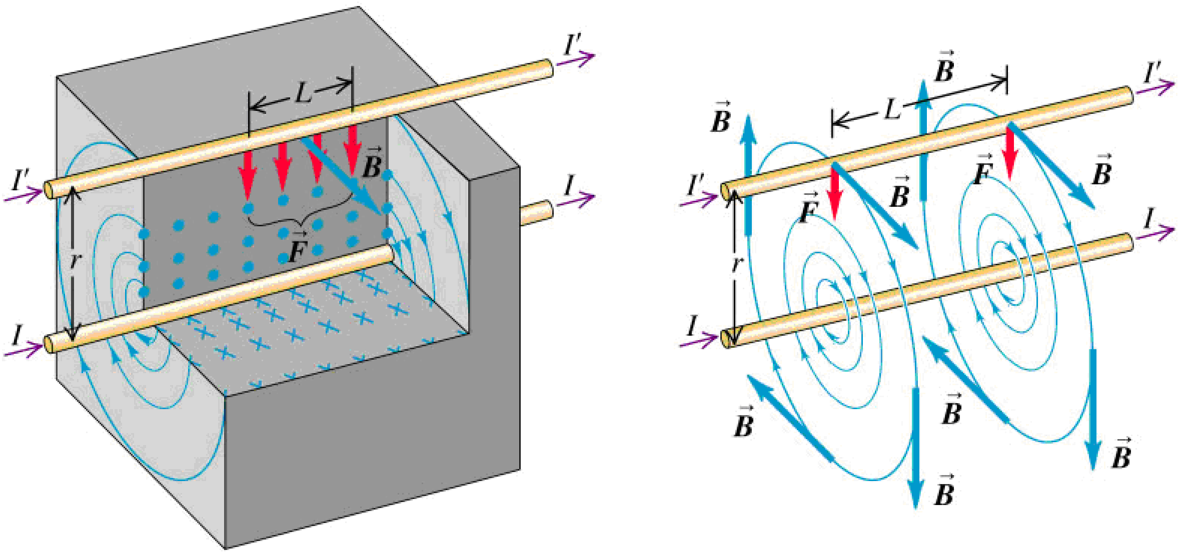
$$= \left(4\pi \times 10^{-7} \frac{T \cdot m}{A}\right)\left(\frac{4.1A}{2\pi(0.032m)}\right)$$

$$= 2.6 \times 10^{-5} T$$

The Ampere as a Unit of Electric Current

We have defined an Ampere as the electric current that transports 1 Coulomb of charge past a given point in a conductor in 1 second. But, while this statement is true, it is not practical (how do we measure how many coulombs of charge pass by a given point. The SI definition of the ampere is a magnetic one and depends on an understanding of the forces between two parallel conductors with a current in each.

Parallel conductors carry currents in the same direction attract each other.



The magnetic field $\vec{B}$ caused by the current in the lower conductor exerts a force $\vec{F}$ on the upper conductor.

Definition: One ampere is that unvarying current that, if present in each of two parallel conductors of infinite length and 1 metre apart in empty space, causes each conductor to experience a force of exactly $2\times10^{-7}N$ per meter of length.

The magnitude of the magnetic field created by wire 1 and experience by wire w, a distance d way is given by

Let $B_1 = u_0\left(\frac{I_1}{2\pi d}\right)$

Then the magnitude of the force acting on wire 2, perpendicular to this field is given by

$$\begin{aligned}F_2 &= I_2 l B_1\\ &= I_2 l u_0\left(\frac{I_1}{2\pi d}\right)\\ &= \frac{u_0 I_1 I_2 l}{2\pi d}\end{aligned}$$

Example

What is the magnitude of the force between two parallel conductors 3.0 m long, with currents of 5.0 A and 9.0 A, 20 cm apart?

Solution:

$$F_2 = \frac{u_0 I_1 I_2 l}{2\pi d}$$

$$= \frac{\left(4\pi \times 10^{-7} \frac{T \cdot m}{A}\right)(5.0A)(9.0A)(3.0m)}{2\pi(0.20m)}$$

$$= 1.4 \times 10^{-4} N$$